Position & Source: Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$

Fundamental Theorems of Vector Calculus:

$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a})$ $\quad$ $\int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \overrightarrow{da}$ $\quad$ $\int (\nabla \times \vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$

Cartesian Coordinates: $\overrightarrow{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ $\quad$ $d\tau = dx\, dy\, dz$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z} \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates: $x = r\, sin\theta\, cos\phi,\ y = r\, sin\theta\, sin\phi,\ z = r\, cos\theta$

$$\overrightarrow{dl} = dr\hat{r} + r\, d\theta\hat{\theta} + r\, sin\theta\, d\phi\hat{\phi} \qquad d\tau = r^2 sin\theta\, dr\, d\theta\, d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\, sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} \qquad \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\, sin\theta}\frac{\partial (sin\theta\, A_\theta)}{\partial \theta} + \frac{1}{r\, sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r\, sin\theta}\left(\frac{\partial (sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (r\, A_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Cylindrical Coordinates: $x = s\, cos\phi,\ y = s\, sin\phi,\ z = z$

$$\overrightarrow{dl} = ds\hat{s} + s\, d\phi\hat{\phi} + dz\hat{z} \qquad d\tau = s\, ds\, d\phi\, dz$$

$$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial (s\, A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s\, A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z} \qquad \nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Dirac Delta Function: $\int \delta^3(\vec{r} - \vec{a})\, d\tau = 1$ if $\vec{a}$ contained in volume, $\delta^3(\overrightarrow{\Delta r}) = \frac{1}{4\pi}\nabla \cdot \left(\frac{\widehat{\Delta r}}{\Delta r^2}\right)$

Irrotational Function (e.g. electrostatic field): $\nabla \times \vec{E} = 0$ $\quad$ $\vec{E} = -\nabla V$ $\quad$ $\oint \vec{E} \cdot \overrightarrow{dl} = 0$
Solenoidal Function (e.g. magnetic field): $\nabla \cdot \vec{B} = 0$ $\quad$ $\vec{B} = \nabla \times \vec{A}$ $\quad$ $\oint \vec{B} \cdot \overrightarrow{da} = 0$

Electric Field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')}{\Delta r^2}\widehat{\Delta r}\, d\tau'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r}')}{\Delta r^2}\widehat{\Delta r}\, da'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{r}')}{\Delta r^2}\widehat{\Delta r}\, dl'$

Magnetic Field: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}') \times \widehat{\Delta r}}{\Delta r^2} d\tau'$, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}') \times \widehat{\Delta r}}{\Delta r^2} da'$, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\vec{r}') \times \widehat{\Delta r}}{\Delta r^2} dl'$

Continuity: $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ (=0 in electrostatics/magnetostatics)

Gauss's Law: $\oint \vec{E} \cdot \overrightarrow{da} = Q_{enc}/\varepsilon_0$, $\nabla \cdot \vec{E} = \rho/\varepsilon_0$, $\oint \vec{D} \cdot \overrightarrow{da} = Q_{f\_enc}$, $\nabla \cdot \vec{D} = \rho_{free}$

Ampere's Law: $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc}$, $\nabla \times \vec{B} = \mu_0 \vec{J}$, $\oint \vec{H} \cdot \overrightarrow{dl} = I_{free\_enc}$, $\nabla \times \vec{H} = \vec{J}_{free}$

Electric Potential: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r'})}{\Delta r} d\tau'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\vec{r'})}{\Delta r} da'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\vec{r'})}{\Delta r} dl'$

$\vec{E} = -\nabla V$, $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot \overrightarrow{dl}$, $\nabla^2 V = -\rho/\varepsilon_0$

Magnetic Vector Potential: $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r'})}{\Delta r} d\tau'$, $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r'})}{\Delta r} da'$, $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r'})}{\Delta r} dl'$

$\vec{B} = \nabla \times \vec{A}$, $\nabla \cdot \vec{A} = 0$ & $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ (Coulomb gauge)

Laplace's Equation: $\nabla^2 V = 0 \; if \; \rho = 0$

Separation of Variables: $\frac{d^2X}{dx^2} = C_1 X, \frac{d^2Y}{dy^2} = C_2 Y, \frac{d^2Z}{dz^2} = C_3 Z$, $C_1 + C_2 + C_3 = 0$, $V = X(x)Y(y)Z(z)$

Separation of Variables (Spherical): $V(r,\theta) = \sum_0^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(cos\theta)$

$P_0(cos\,\theta) = 1, P_1(\cos\theta) = \cos\theta, P_2(\cos\theta) = \frac{3cos^2\theta - 1}{2}$

Multipoles: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_0^\infty \frac{1}{r^{n+1}} \int (r')^n P_n(cos\alpha) \rho(\vec{r}') d\tau'$, $\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \sum_0^\infty \frac{1}{r^{n+1}} \int (r')^n P_n(cos\alpha) \overrightarrow{dl'}$

Dipoles: $\vec{p} = \int \vec{r}' \rho(\vec{r}') d\tau'$, $V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$, $\vec{\tau} = \vec{p} \times \vec{E}$, $\vec{F} = (\vec{p} \cdot \nabla)\vec{E}$, $\vec{P} = \vec{p}/volume$

$\vec{m} = I\vec{a}$, $\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$, $\vec{\tau} = \vec{m} \times \vec{B}$, $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$, $\vec{M} = \vec{m}/volume$

Fields in Matter: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$, $\sigma_b = \vec{P} \cdot \hat{n}$, $\rho_b = -\nabla \cdot \vec{P}$

$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$, $\vec{K}_b = \vec{M} \times \hat{n}$, $\vec{J}_b = \nabla \times \vec{M}$

Linear Materials: $\vec{P} = \varepsilon_0 \chi_e \vec{E}$, $\vec{D} = \varepsilon \vec{E} = (1 + \chi_e)\varepsilon_0 \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$

$\vec{M} = \chi_m \vec{H}$, $\vec{B} = \mu \vec{H} = (1 + \chi_m)\mu_0 \vec{H}$

Boundary Conditions: $\Delta V = 0$, $\Delta \vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n} = -\Delta \left(\frac{\partial V}{\partial n}\right) \hat{n}$, $\Delta D_\perp = \sigma_f$, $\Delta \vec{D}_{||} = \Delta \vec{P}_{||}$

$\Delta \vec{A} = 0$, $\Delta \vec{B} = \mu_0(\vec{K} \times \hat{n})$, $\Delta \left(\frac{\partial \vec{A}}{\partial n}\right) = -\mu_0 \vec{K}$, $\Delta \vec{H}_{||} = \vec{K}_{free} \times \hat{n}$, $\Delta H_\perp = -\Delta M_\perp$

Lorentz Force: $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$, On Wire: $\vec{F}_{mag} = \int I(\overrightarrow{dl} \times \vec{B})$

Work and Energy: $W = Q\Delta V$, $W_{electrostatic} = \frac{\varepsilon_0}{2} \int E^2 d\tau$, $W_{elec+polarization} = \frac{1}{2} \int \vec{E} \cdot \vec{D}\, d\tau$